Analytical and Experimental Investigations on Several Resonant Modes in Open Dielectric Resonators

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Abstract —**The complex resonant frequency of open dielectric pillbox resonators is analyzed by an analytical method proposed by the present authors, which expands the field into a truncated series of solutions of the Helmholtz equation in the spherical coordinates and treats the boundary condition in the least-squares sense. This method is applied to calculate the characteristics of several resonant modes which will be of practical use. The accuracy of the method is confirmed by investigating the convergence of solutions. Also, numerical results are compared with experimental results of several resonant modes, which are obtained for the dielectric samples with $\epsilon_r = 38.0$ and 19.5 in the X-band.**

I. Introduction

Dielectric pillbox resonators of the open type have found many practical applications, particularly in the spectral range from microwave to short millimeter-wave frequencies [1]–[8]. Nevertheless, there are few methods of effective use in calculating the complex resonant frequency; the resonant frequency and the Q-factor due to radiation loss, for arbitrary permittivity ϵ_r. One effective method used to analyze them has been presented by Van Bladel *et al.* [9]–[11]. Their approach is based on the asymptotic expansion of fields in powers of the reciprocal of $\sqrt{\epsilon_r}$, so that the validity of their method is limited to the case of relatively high ϵ_r, say 100 [11]. For improving the accuracy of their method, it will be necessary to introduce several higher order terms in $1/\sqrt{\epsilon_r}$.

An alternative method has been developed by the present authors [12]. Their approach, based on the Rayleigh expansion theorem, analyzes the complex resonant frequency without a limit on ϵ_r and is accurate in the sense that the complex resonant frequency converges to the exact value as the number of terms in the truncated expansion increases. However, the authors have shown the numerical results for $TE_{01\delta}$ and $TM_{01\delta}$ modes only and also have had no experimental discussion.

The purpose of this paper is to show numerically the complex resonant frequency for several resonant modes which will be of practical use, and also to discuss experimentally the resonant characteristics of several modes, including hybrid modes, along with the numerical results obtained by the present method.

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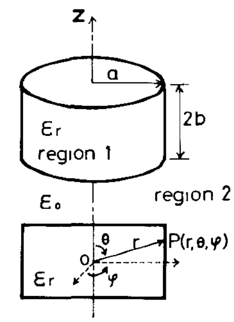


Fig. 1. Dielectric pillbox resonator and spherical coordinate system.

II. Analysis

A. Formulation of the Problem

Fig. 1 shows the geometry of an isolated dielectric pillbox resonator which is surrounded by the medium with the relative permittivity ϵ_0. The radius of the pillbox is a, the thickness is $2b$, and the relative permittivity is ϵ_r. Our general method easily analyzes the complex resonant frequency of higher order resonant modes, as well as the lowest φ-independent modes without any complexity. Reference [12] describes in detail the method, but a brief summary is in order here.

First, we expand the fields in region 1 and region 2 in terms of solutions to the Helmholtz equation in the spherical coordinate system (r, θ, φ) obtained by separation of variables. By referring to [12, eq. (1)] or [13, eqs. (6)–(26)], the fields of a resonator at an arbitrary angular frequency ω can be expressed by the following scalar potentials Ψ_{ri} and $\bar{\psi}_{ri}$ $(i=1,2)$, which generate a field TM to r and a field TE to r, respectively:

$$\psi_{ri} = \cos(m\varphi + \varphi_0)\sum_n A_{ni}\sqrt{k_i r}\, F_{n+1/2}(k_i r)\, P_n^m(\cos\theta)\, e^{j\omega t}$$

$$\bar{\psi}_{ri} = \sin(m\varphi + \varphi_0)\sum_n \bar{A}_{ni}\sqrt{k_i r}\, F_{n+1/2}(k_i r)\, P_n^m(\cos\theta)\, e^{j\omega t}$$

$$(i=1,2) \quad (1)$$

where A_{ni} and $\bar{A}_{ni}$ are modal expansion coefficients to be determined, φ_0 is an arbitrary phase angle, and k_i is the wavenumber in the region $(i=1,2)$. $P_n^m(\cos\theta)$ is the first-kind associated Legendre function of order n, m, and

$F_{n+1/2}(k_i r)$ is given by

$$F_{n+1/2}(k_i r) = \begin{cases} J_{n+1/2}(k_1 r), & \text{for region 1} \\ H^{(2)}_{n+1/2}(k_2 r), & \text{for region 2} \end{cases} \tag{2}$$

where $J_{n+1/2}$ and $H^{(2)}_{n+1/2}$ are the first kind of the Bessel function and the second kind of the Hankel function of the order $n+1/2$, respectively.

The characteristic angular resonant frequency (complex value) $\Omega = \Omega_r + j\Omega_i$ is determined by considering the boundary condition on the resonator surface. However, the infinite series in (1) should be truncated to a finite number of terms $n = N$ in practical calculations. Such approximated fields are therefore fitted to the boundary condition in the least-squares sense [14]. Since the geometry of the resonator under consideration has axial symmetry with respect to the z-axis, the mean-squares error E in the boundary condition can be written by the following line integral [12]:

$$E = \int_\Gamma \{ |\mathbb{E}_{t1} - \mathbb{E}_{t2}|^2 + Z^2 |\mathbb{H}_{t1} - \mathbb{H}_{t2}|^2 \} \, dl \tag{3}$$

where Γ denotes the boundary contour on the $r-\theta$ plane (but $0 \leqslant \theta \leqslant \pi$) at an arbitrary φ coordinate, $\mathbb{E}_{ti}, \mathbb{H}_{ti} (i = 1,2)$ denote the field components tangential to Γ, and the intrinsic impedance of the region 1 ($\sqrt{\mu/\epsilon_0\epsilon_r}$) is used for an arbitrary impedance parameter Z. Since the resonator shown in Fig. 1 has a plane of symmetry with respect to the $r-\varphi$ plane at $\theta = \pi/2$, the calculation of (3) becomes rather simple by taking the integral contour only in $0 \leqslant \theta \leqslant \pi/2$, along with a simple relation of $P_n^m(\cos\theta)$ at $\theta = \pi/2$ [12].

Now, minimizing E with respect to both the modal coefficients and the angular frequency ω, we obtain the characteristic angular resonant frequency $\Omega = \Omega_r + j\Omega_i$ ($\Omega_r > 0, \Omega_i > 0$) by the same procedure as described in [14]. This complex quantity Ω explicitly leads to both the resonant frequency f_0 and the intrinsic Q value Q_0 due to radiation loss through the following relations:

$$f_0 = |\Omega|/2\pi = k_0 c/2\pi \quad Q_0 = |\Omega|/2\Omega_i \tag{4}$$

where k_0 is the free-space wavenumber corresponding to the resonant frequency f_0 and c is the velocity of light in free space. The method mentioned here assures mathematically the uniform convergence in the sequence of the truncated modal expansions such as in (1) [15].

B. Numerical Results

Apart from the analytical treatment mentioned in the previous section, how to classify the resonant modes will be followed here by way of classifying modes in a cylindrical resonator [5], [12]. Throughout this section, the calculation will be performed for the structure with $b/a = 1.0$, and ϵ_0 is put as unity.

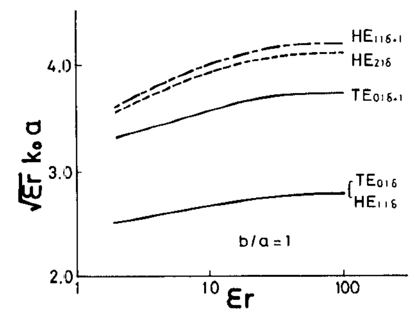
First, we compute both the normalized resonant frequency $k_0 a$ and the intrinsic Q value Q_0 of the $HE_{ml\nu}$ modes including the $TE_{01\delta}$ mode. As commonly known, resonant modes of this group are characterized by the predominant magnetic field in the z-direction. We have already investigated the convergence of both $k_0 a$ and Q_0 for the $TE_{01\delta}$ mode [12]. For convenience sake, those results are shown again in Table I, along with Table II, which shows the similar calculations for the hybrid $HE_{11\delta}$ mode. Both $k_0 a$ and Q_0 of Tables I and II manifest a good convergence for $N \geqslant 5$, though these tables show the results obtained only for $\epsilon_r = 35$.

TABLE I
NORMALIZED RESONANT FREQUENCIES AND INTRINSIC Q VALUES OF THE $TE_{01\delta}$ MODE CALCULATED FOR THE DIFFERENT NUMBER N OF THE EXPANSION TERMS ($\epsilon_r = 35$, $b/a = 1$).

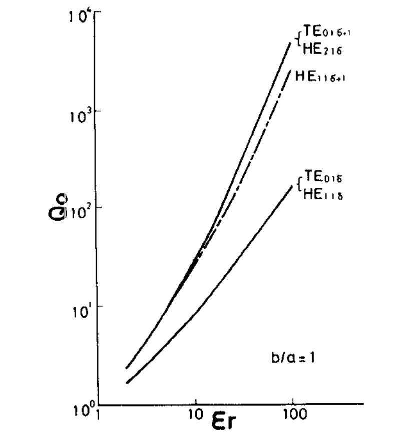
N	$k_0 a$	Q_0
1	0.474	44.3
2	0.473	43.3
3	0.470	40.1
4	0.469	40.0
5	0.467	39.5
6	0.467	39.5
7	0.467	39.3
8	0.467	39.3

TABLE II
NORMALIZED RESONANT FREQUENCIES AND INTRINSIC Q VALUES OF THE $HE_{11\delta}$ MODE CALCULATED FOR THE DIFFERENT NUMBER N OF THE EXPANSION TERMS ($\epsilon_r = 35$, $b/a = 1$).

N	$k_0 a$	Q_0
1	0.475	44.3
2	0.473	41.0
3	0.467	39.4
4	0.467	39.5
5	0.467	39.2
6	0.467	39.2
7	0.467	39.2
8	0.467	39.2



(a)



(b)

Fig. 2. Resonant characteristics of the HE-mode group as a function of ϵ_r. (a) Normalized resonant frequency. (b) Intrinsic Q value.

TABLE III
NORMALIZED RESONANT FREQUENCIES AND INTRINSIC Q VALUES OF THE $TM_{01\delta}$ MODE CALCULATED FOR THE DIFFERENT NUMBER N OF THE EXPANSION TERMS ($\epsilon_r = 35$, $b/a = 1$).

N	k₀a	Q₀
5	0.671	26.7
6	0.670	26.3
7	0.670	24.8
8	0.669	24.6
9	0.669	23.8
10	0.669	23.7
11	0.669	23.2
12	0.669	23.2

TABLE IV
NORMALIZED RESONANT FREQUENCIES AND INTRINSIC Q VALUES OF THE $EH_{11\delta}$ MODE CALCULATED FOR THE DIFFERENT NUMBER N OF THE EXPANSION TERMS ($\epsilon_r = 35$, $b/a = 1$).

N	k₀a	Q₀
5	0.630	40.0
6	0.632	41.9
7	0.631	40.4
8	0.632	41.1
9	0.632	40.3
10	0.632	40.7
11	0.632	40.2
12	0.632	40.2

For a hybrid mode, N means the number of expansion terms of each of ψ_{ri} and $\bar{\psi}_{ri}$ in (1). As a result, it will be enough to take $N = 10$ for accurate calculations for the HE-mode group, and Fig. 2(a) and (b) shows $\sqrt{\epsilon_r}\, k_0 a$ and Q_0 for several resonant modes, as a function of ϵ_r.

Next, Tables III and IV show the similar calculations relating to the convergence for the $TM_{01\delta}$ mode and the hybrid $EH_{11\delta}$ mode, respectively. We see here that the convergence for the EH-mode group is slower than that for the HE-mode group.

It is well known that the edge-shaped boundaries as seen in Fig. 1 usually cause the slow convergence in actual calculations, although the method is complete in theory. The EH-mode group has a predominant electric field in the z-direction. This electric field transverse to the resonator's edges may be singular [16]. So, we may understand that the dielectric edges in the resonator under consideration cause a significantly slow convergence for the EH-mode group. Indeed, Tables III and IV show that the calculated results almost converge for $N \geqslant 11$, about twice as large as that of the HE-mode group. Hence, for the EH-mode group, both $\sqrt{\epsilon_r}\, k_0 a$ and Q_0 are calculated with $N = 16$, and the results are shown in Fig. 3(a) and (b) as a function of ϵ_r.

III. EXPERIMENTS

A. Experimental Setup

The experimental setup in the X-band is shown schematically in Fig. 4. The microwave oscillator used can sweep the frequency range 6.5–12.4 GHz. In the experiments, five dielectric samples are used for pillbox resonators. The samples I, II, III, and IV have the same relative permittivity $\epsilon_r = 38.0$ and have, respectively, the following dimensions: $2a \times 2b = 5.72$ mm $\times 2.38$ mm, 5.72×2.40, 5.79×2.40, and 5.79×2.42; the structural ratio $b/a \simeq 0.42$ is almost the same for these samples. One more sample (sample V) has the parameters: $\epsilon_r = 19.5$, $2a \times 2b = 9.01$ mm $\times 8.53$ mm ($b/a \simeq 0.95$).

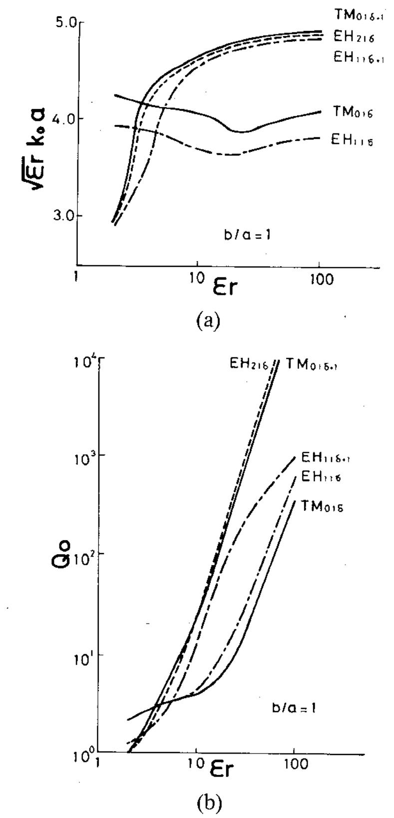


Fig. 3. Resonant characteristics of the EH-mode group as a function of ϵ_r. (a) Normalized resonant frequency. (b) Intrinsic Q value.

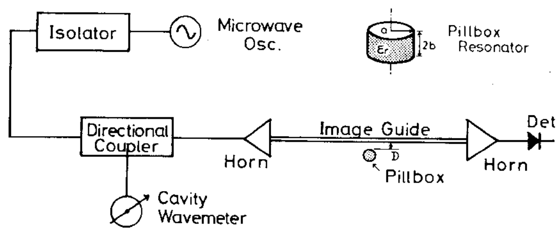


Fig. 4. Experimental setup for investigating the resonant characteristics in the X-band.

For exciting a resonant mode in a pillbox, one can use a rectangular dielectric image line which is put side by side with a pillbox. In our experiments, both waveguide and pillbox are put on a metal plate having the area 0.5×1 m^2. First, we utilize the samples I–IV for investigating the resonance of TE modes. In these samples, the resonances take place only for three modes: $TE_{01\delta}$, $TE_{01\delta+1}$, and $HE_{11\delta}$, in the above frequency range. To excite these modes in a resonator, a TE propagating mode mainly polarized parallel to the metal plate is launched in the image line, and the coupling gap D is kept large enough to have a small coupling.

For the $TE_{01\delta+1}$ and the $HE_{11\delta}$ modes, one may replace the $r-\varphi$ plane at $\theta = \pi/2$ in Fig. 1 with a short-circuited plane, so that the metal plate has no effect on the resonant

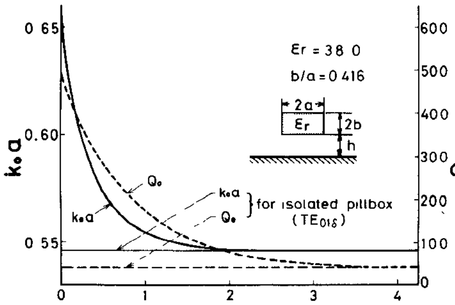


Fig. 5. Effect of the metal plate on the resonant characteristics of the $TE_{01\delta}$ mode.

characteristics of these modes. However, the $HE_{11\delta}$ mode is so sensitive to the air gap between the metal plate and the pillbox that it is quite difficult to get responsible data, and we do not investigate the characteristics of this mode experimentally. It should be noted here that a sample put on the metal plate is regarded as an isolated pillbox having the thickness $2b$ twice as thick as that of the original sample (i.e., $b/a \simeq 0.84$).

On the other hand, the $TE_{01\delta}$ mode can replace the $r-\varphi$ plane at $\theta = \pi/2$ with an open-circuited plane, so that it is impossible to realize an isolated pillbox by putting a sample directly on the metal plane. Hence, in our experiments, a pillbox is elevated upward by the height h from the metal plate by means of a slender rod of foamed polystyrene (2 mmϕ, $\epsilon_r \simeq 1.02$). Neglecting the effect of the polystyrene rod, both k_0a and Q_0 are calculated as a function of h/a, as shown in Fig. 5. It is found that the resonant frequency is almost the same as that of an isolated pillbox if $h/a > 2$, but the Q value becomes slightly larger even at $h/a = 3$. So, the experiments for the $TE_{01\delta}$ mode are performed at $h/a = 3$, and the numerical results in the following section are calculated by considering $h/a = 3$.

Next, sample V is utilized for investigating the resonant characteristics of hybrid modes. Unlike the samples mentioned above, this sample shows the resonances of $TE_{01\delta+1}$, $HE_{11\delta+1}$, $HE_{21\delta}$ $TM_{01\delta}$, $EH_{11\delta}$, $EH_{11\delta+1}$, and $EH_{21\delta}$ modes in the frequency range of our sweep oscillator. In this case, the sample is always set with a height from the metal plate by using a polystyrene rod, and a TE mode or a TM mode propagating in the image line is used to excite selectively the HE-mode group or the EH-mode group in the resonator.

B. Experimental Results

Fig. 6 shows a typical resonant curve of the $TE_{01\delta}$ mode obtained for sample I. In our experiments, the intrinsic Q value is obtained from the best-fitted Lorentzian for the measured curve by assuming that the coupling between waveguide and pillbox is small enough and the adjacent resonances interfere little with each other. To confirm the latter point, both k_0a and Q_0 are calculated as a function of the structural ratio b/a, as shown in Fig. 7. It is found that the $TE_{01\delta}$ and the $HE_{11\delta}$ modes almost degenerate at $b/a = 1$, but decreasing b/a significantly splits this degenerate. As mentioned before, all of the samples have almost

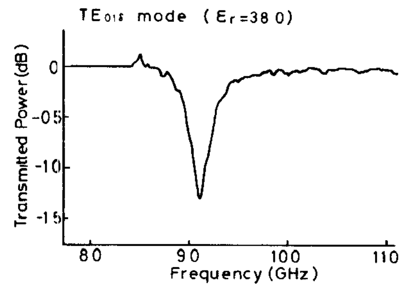


Fig. 6. Typical resonant curve of the $TE_{01\delta}$ mode obtained for sample I ($b/a = 0.416$).

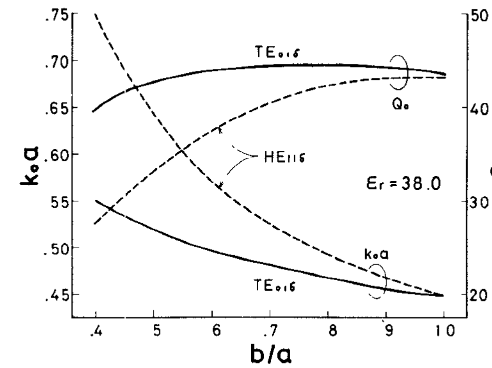


Fig. 7. Resonant characteristics of the $TE_{01\delta}$ and the $HE_{11\delta}$ modes as a function of the structural ratio b/a.

TABLE V
COMPARISON BETWEEN THE MEASURED AND THE CALCULATED RESULTS OF BOTH RESONANT FREQUENCIES AND Q VALUES FOR THE $TE_{01\delta}$ MODE ($\epsilon_r = 38.0$).

Sample	Resonant Frequency (GHz)		Q value	
	measured	theoretical	measured	theoretical
I	9.11	9.13	46	50.1
II	9.10	9.10	46	50.2
III	9.05	9.04	45	50.1
IV	9.00	9.01	49	50.1

the same ratio $b/a \simeq 0.42$ for the resonance of the $TE_{01\delta}$ mode. Let us consider here sample I, for example, which has the ratio $b/a = 0.416$. Fig. 7 shows that this resonator has the $TE_{01\delta}$ mode as the resonant mode of the lowest order and the $HE_{11\delta}$ mode becomes the next higher order mode. From Fig. 7, the resonant frequency of the $TE_{01\delta}$ mode is found to be 9.13 GHz ($k_0a = 0.547$), while that of the $HE_{11\delta}$ mode is found to be 12.15 GHz ($k_0a = 0.728$), which is entirely beyond the frequency range of Fig. 6. Moreover, as mentioned before, we may expect the selective excitation of TE modes in a pillbox through an image line. Hence, we may conclude that there is no adjacent mode interfering with the resonance of the $TE_{01\delta}$ mode, and the resonant curve of Fig. 6 is of the $TE_{01\delta}$ mode itself. We have investigated the effect of interference among adjacent modes not only for the $TE_{01\delta}$ mode in the other samples, but also for the $TE_{01\delta+1}$ modes, and have confirmed that no interference occurs. Tables V and VI summarize the measured resonant frequencies and the Q values for the $TE_{01\delta}$ mode and the $TE_{01\delta+1}$ mode, respectively. It is found that the measured resonant frequencies

TABLE VI
COMPARISON BETWEEN THE MEASURED AND THE CALCULATED RESULTS OF BOTH RESONANT FREQUENCIES AND Q VALUES FOR THE $TE_{01\delta+1}$ MODE ($\epsilon_r = 38.0$).

Sample	Resonant Frequency(GHz)		Q value	
	measured	theoretical	measured	theoretical
I	10.89	10.90	420	482
II	10.86	10.86	410	483
III	10.80	10.79	420	481
IV	10.73	10.75	410	482

TABLE VII
COMPARISON BETWEEN THE MEASURED AND THE CALCULATED RESULTS OF BOTH RESONANT FREQUENCIES AND Q VALUES FOR SEVERAL HYBRID MODES ($\epsilon_r = 19.5$).

Mode	Resonant Frequency(GHz)		Q value	
	measured	theoretical	measured	theoretical
$EH_{11\delta}$	9.04	9.04	18	11
$TE_{01\delta+1}$	9.03	9.04	130	110
$TM_{01\delta}$	——	9.44	——	8
$HE_{21\delta}$	9.80	9.73	110	110
$HE_{11\delta+1}$	10.03	10.00	110	112
$EH_{11\delta+1}$	11.53	11.59	89	96
$EH_{21\delta}$	11.74	11.73	157	166

agree well with the calculated ones, while the agreement between Q values is somewhat poor. Such a discrepancy, about 15-percent maximum, will be unavoidable because of less accuracy in the Q measurement in our experimental procedure, especially because of a lack of considering the external Q value.

Table VII indicates the results obtained for sample V. As the $TM_{01\delta}$ mode in this sample has a quite low Q value, we cannot measure both f_0 and Q_0. Some modes, in this case, show a little interference with each other, and the confidence in measured data, especially for Q values, is slightly worse than that obtained for the other samples.

Nevertheless, the experimental results in this section will conclude that the analytical method [12] is effective in practice to calculate the Q value, as well as the resonant frequency, of a pillbox resonator having arbitrary permittivity.

IV. CONCLUSION

First, the analytical method for an open dielectric pillbox resonator, previously proposed by the authors, has been applied to analyze several resonant modes which would be of practical use. The accuracy of the method was confirmed by investigating the convergence of calculations for $TM_{01\delta}$ and $EH_{11\delta}$ modes, as well as $TE_{01\delta}$ and $HE_{11\delta}$ modes. As expected from the effect at the dielectric edges of a resonator, it was found that the convergence for the EH-mode group was slower than that for the HE-mode group.

Next, the experiments have been performed for the samples with $\epsilon_r = 38.0$ and 19.5 in the X-band, and it was confirmed that the calculated results have sufficiently explained the experimental results of both the resonant frequency and the Q value.

However, some problems still remain to be solved. One of them will be to reduce the radiation loss for a resonant mode which will be of practical use. A method will be discussed in a succeeding paper.

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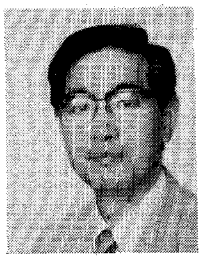

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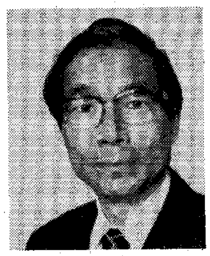

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Analysis of Hybrid Field Problems by the Method of Lines with Nonequidistant Discretization

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Abstract —**The method of lines, which has been proved to be very efficient for calculating the characteristics of one-dimensional and two-dimensional planar microwave structures, is extended to nonequidistant discretization. By means of an intermediate transformation it is possible to maintain all essential transformation properties that are given in the case of equidistant discretization. The flexibility of the method of lines is increased substantially. As a consequence, the accuracy is improved with reduced computational effort.**

I. Introduction

A SUCCESSFUL DESIGN of planar microwave circuits presupposes accurate knowledge of the characteristics of the elementary components.

In principle, an exact determination of the characteristics of passive components like transmission lines, resonators, and filters is possible by means of complete Fourier series expansions. For numerical evaluation, only a finite number of terms can be taken into account. Hence, this method is characterized by the fact that the exactly formulated problem is solved approximately.

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A completely different way is taken by the grid-point method and the method of lines [1], where the approximately formulated problem is solved exactly.

The semi-analytical method of lines has been applied to various problems of physics [2]. An essential extension of this method is given in [3] for the one-dimensional and in [4] for the two-dimensional hybrid problem of planar waveguides. It has been shown that this class of waveguides can be solved accurately and in a simple manner.

In the limiting case of an infinite number of lines, exactly the same solution is obtained as in the limiting case of an infinite number of terms in the Fourier series expansions.

The relative convergence phenomenon, which is a consequence of the Fourier series truncations, does not occur with the method of lines. Optimum convergence is always assured, if the simple condition is satisfied that the strip-edges are located at definite positions with respect to the adjacent ψ^e- and ψ^h-lines [5]. It should be noted, however, that the convergence of the propagation constant, the characteristic impedance or the resonant frequency does not critically depend on the edge parameters, so that the problem of convergence on the whole is not critical.

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